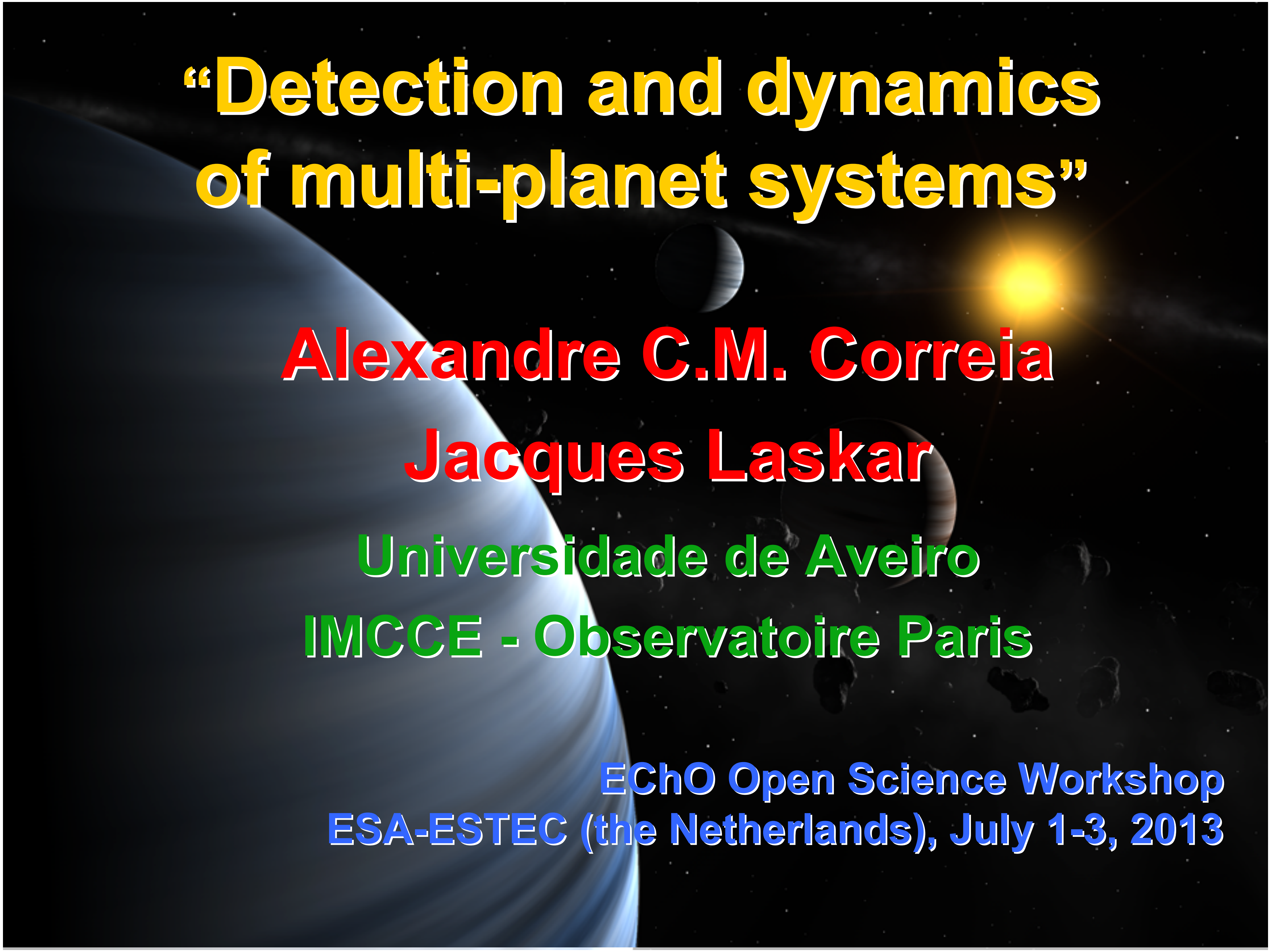
"Detection and dynamics of multi-planet systems"
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IMCCE - Observatoire Paris
EChO Open Science Workshop
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Two planets motion

$$\ddot{\vec{r}} = -G\frac{(m+M)}{r^3}\vec{r} + \vec{\nabla}_{\vec{r}} R$$

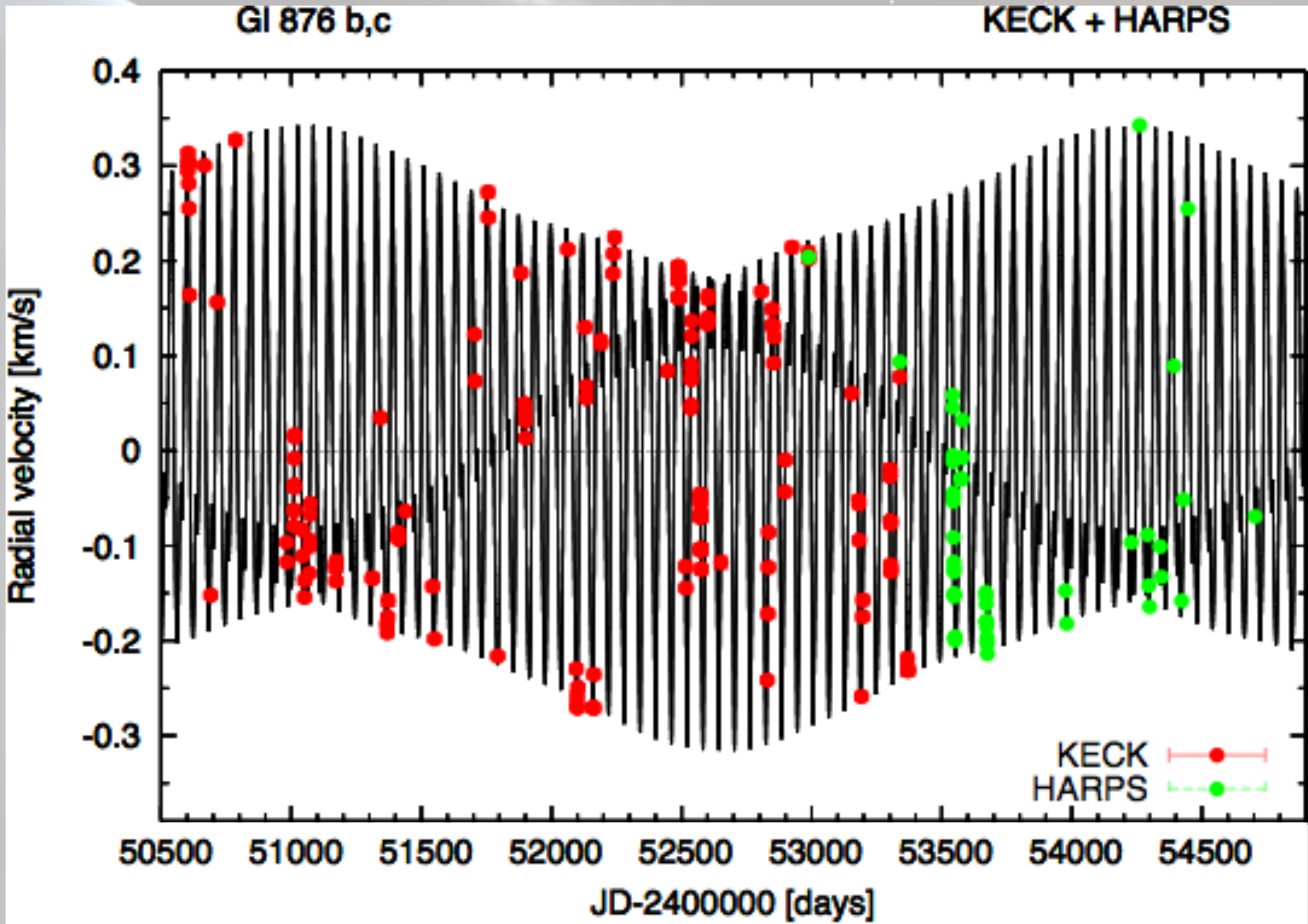
Disturbing function:

$$R = Gm'\left(\frac{1}{|\vec{r}\,'-\vec{r}|} - \frac{\vec{r}\cdot\vec{r}\,'}{r'^3}\right)$$

Since *m'* << *M*, the disturbing function can be seen as a perturbation of the keplerian motion.

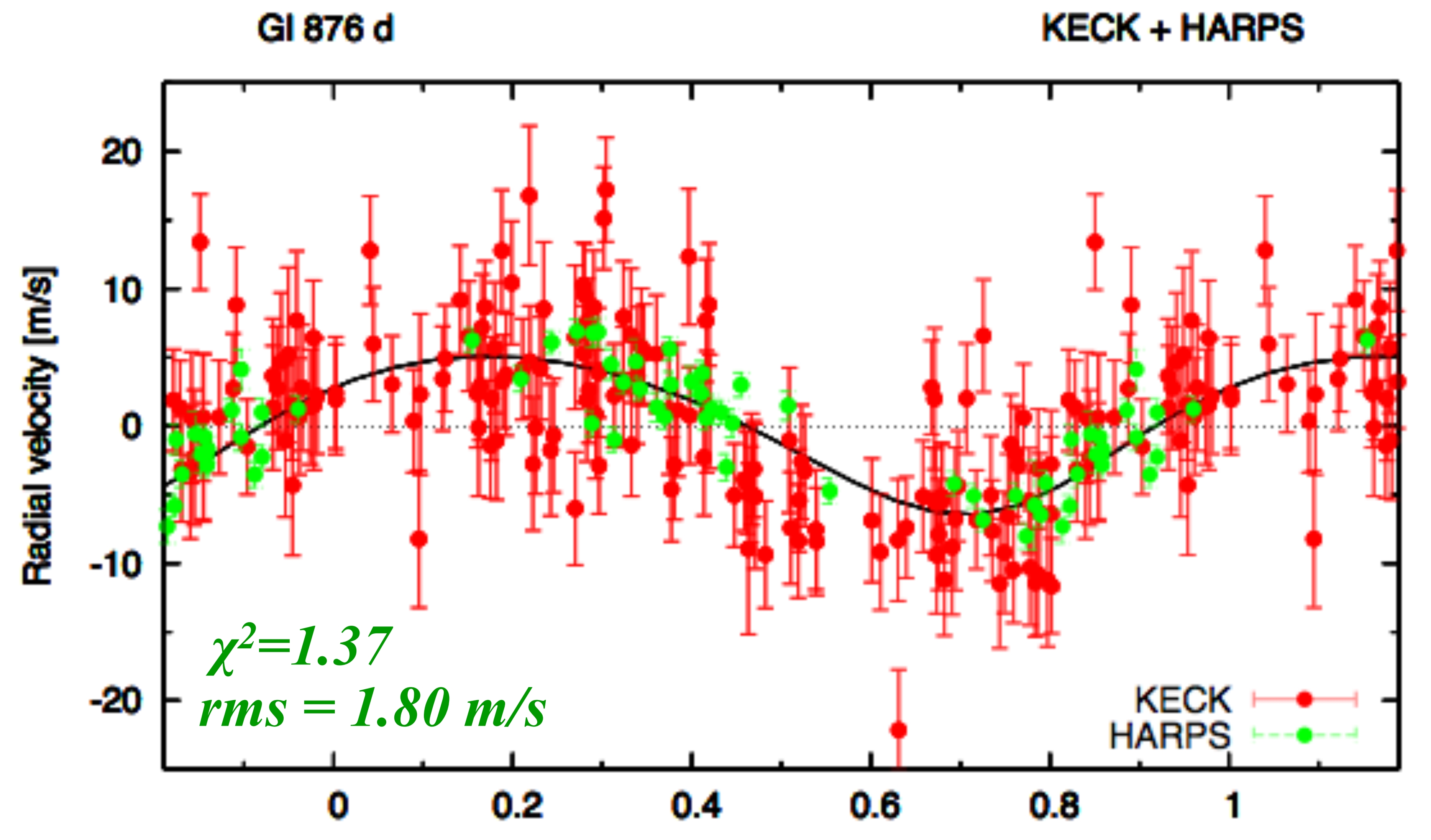
GJ 876, a "case study"

χ^2=2.60 $i = 49^o \pm 1^o$ rms = 4.45 m/s



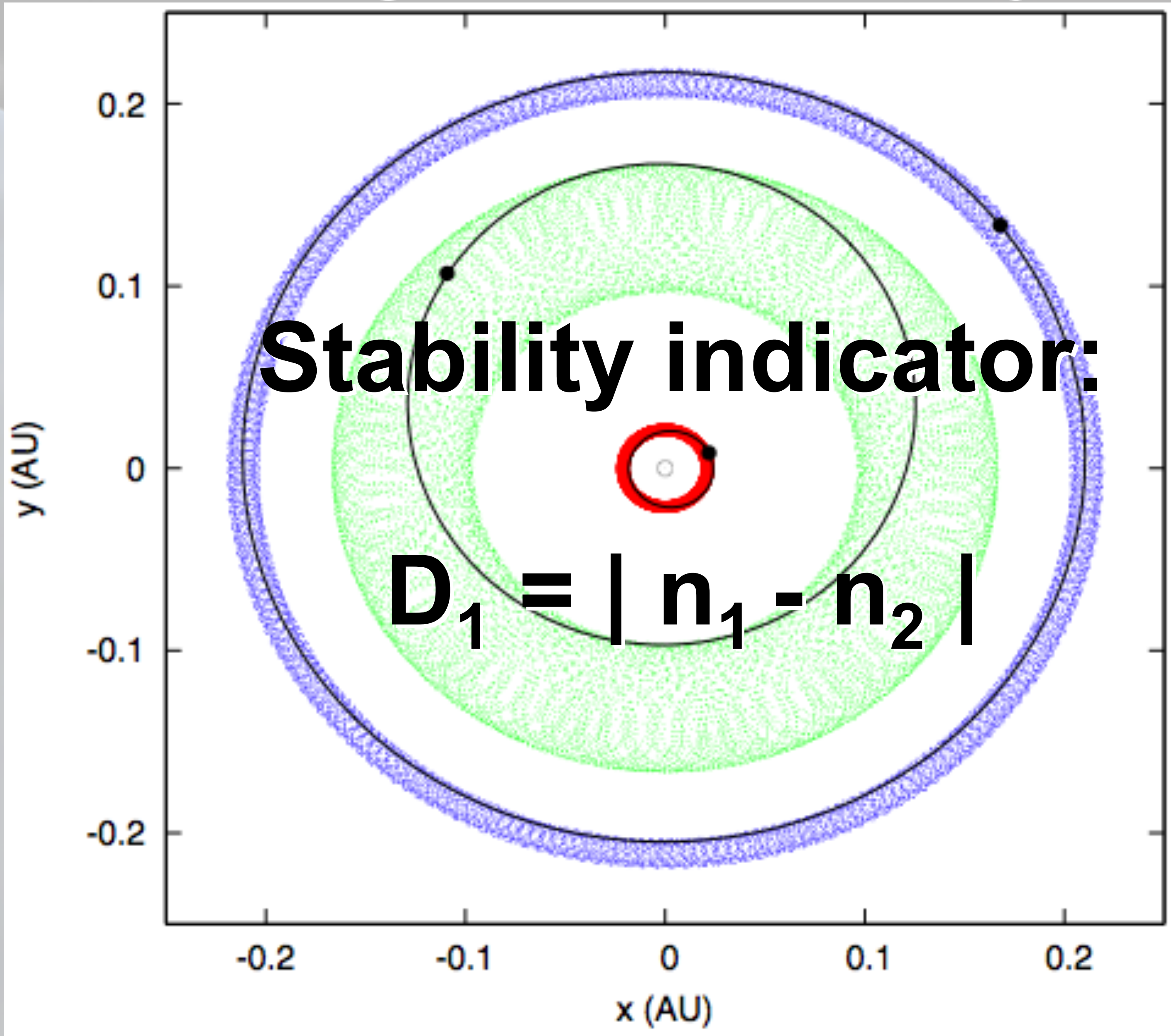
Correia et al. (A&A, 2010)

3^{rd} planet with 6 M_E



Correia et al. (A&A. 2010)

Long-term stability

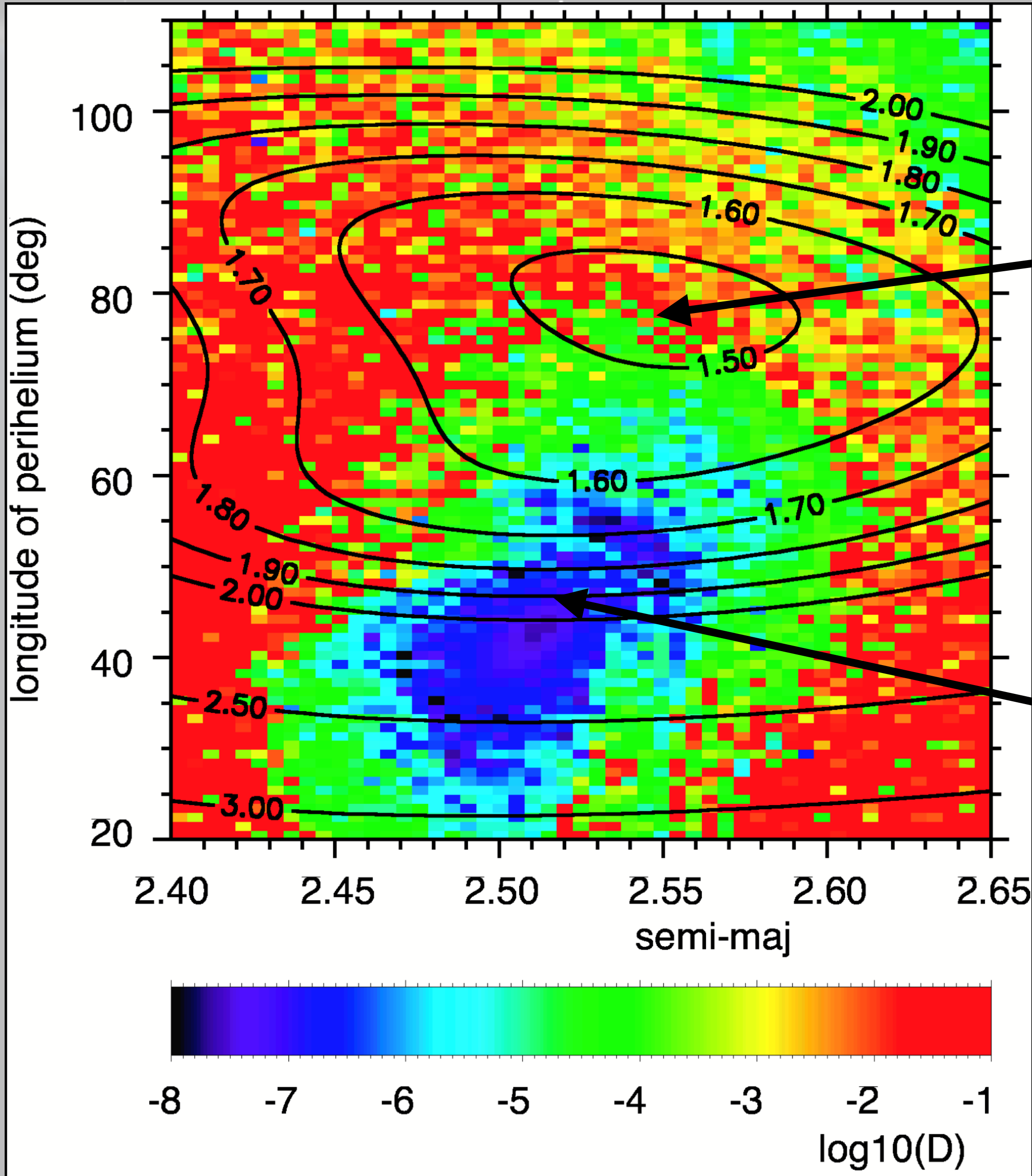


Stability indicator:

$$D_1 = | n_1 - n_2 |$$

Correia et al. (A&A. 2010)

HD202206 (5:1 resonance, 2005 data)



a = 2.55 AU

ω = 79.0°

χ^2=1.47

rms = 9.65 m/s

a = 2.54 AU

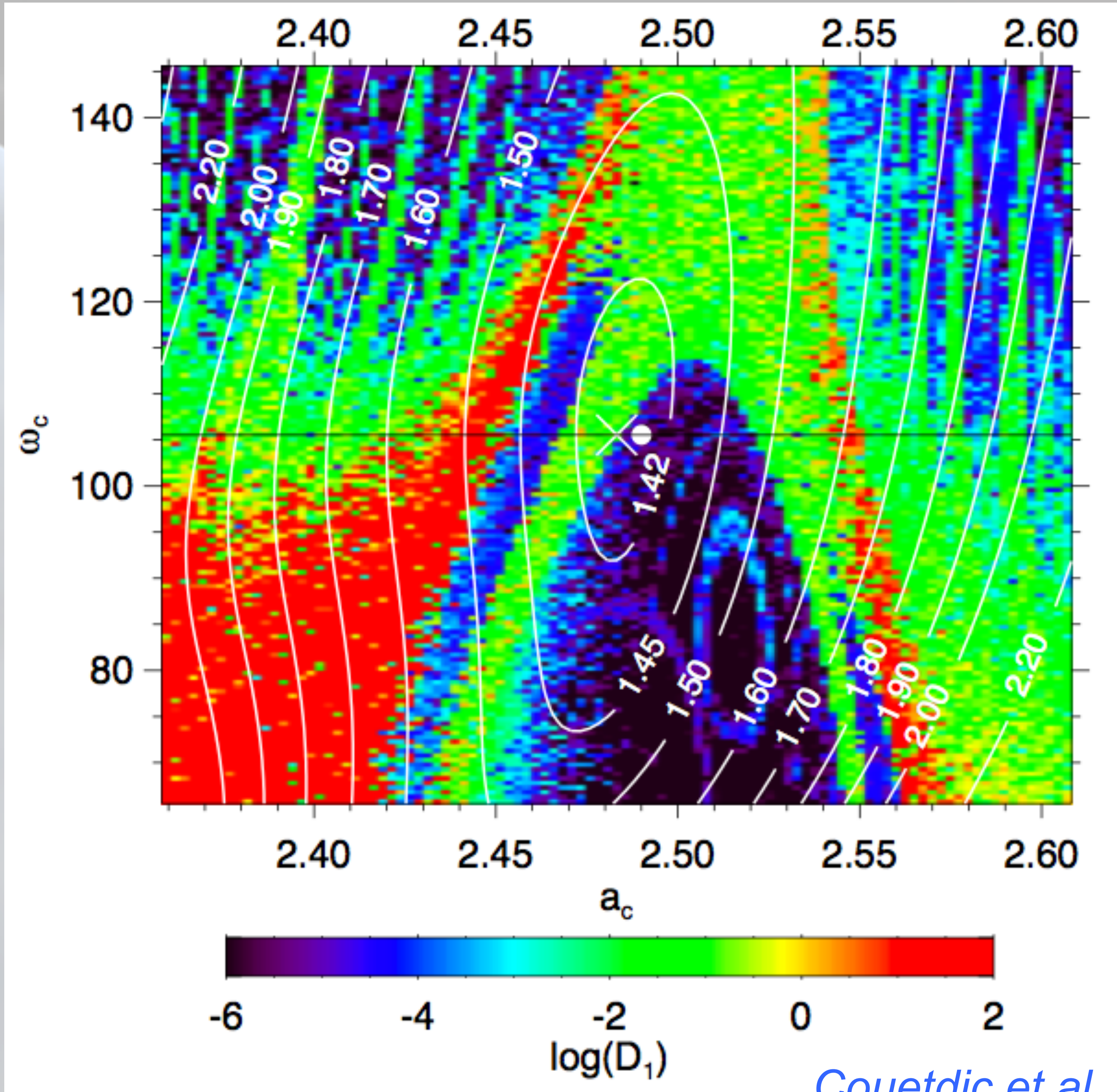
ω = 55.5°

χ^2=1.67

rms = 10.7 m/s

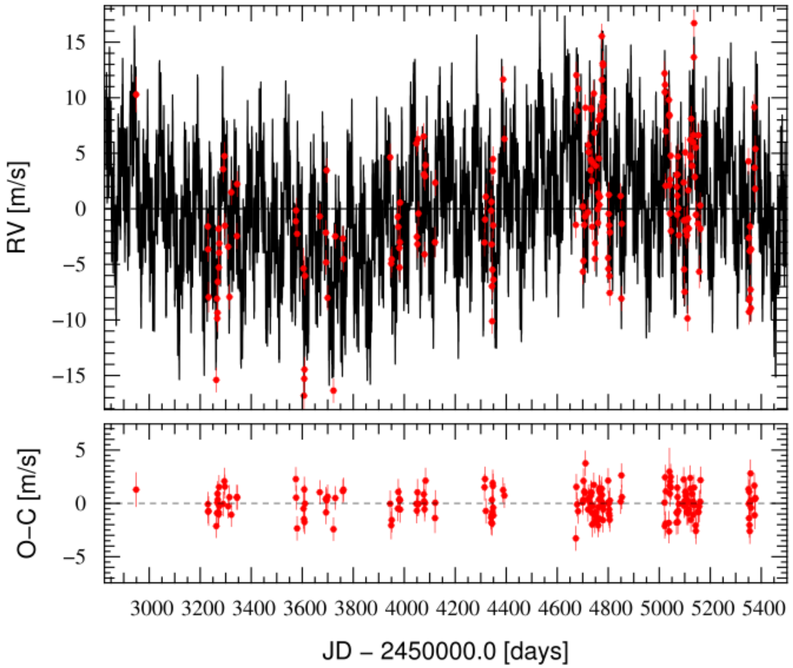
Correia et al. (2005)

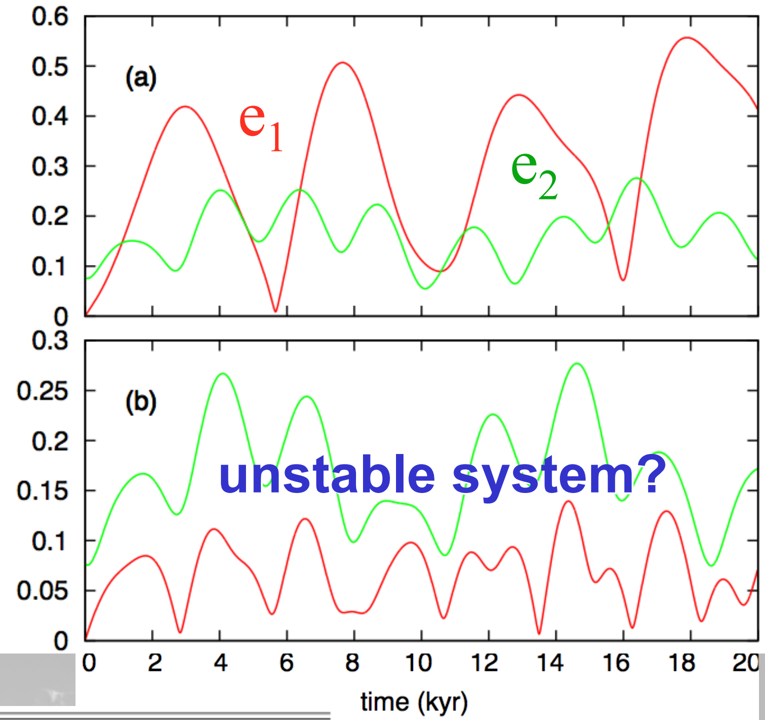
HD202206 (2010 updated data)



Couetdic et al. (A&A 2010)

HD 10180, seven planets!





Parameter	[unit]	HD 10180 b	HD 10180 c	HD 10180 d	HD 10180 e	HD 10180 f	HD 10180 g	HD 10180 h
Epoch	[BJD]				2,454,477.878676 (fixed)			
i	[deg]				90 (fixed)			
V	[km s^{-1}]				35.53014(±0.00045)			
P	[days]	1.177662 (±0.000090)	5.75962 (±0.00029)	16.3570 (±0.0042)	49.747 (±0.023)	122.72 (±0.19)	602 (±11)	2229 (±106)
	[deg]	142 (±11)	29.4 (±1.9)	99.4 (±3.3)	20.9 (±2.2)	237.8 (±3.2)	253 (±11)	317.6 (±4.1)
e		0.0 (fixed)	0.077 (±0.032)	0.142 (±0.060)	0.061 (±0.036)	0.127 (±0.066)	0.0 (fixed)	0.145 (±0.073)
	[deg]	0.0 (fixed)	−41 ($^{+70}_{-141}$)	−51 ($^{+43}_{-10}$)	171 (±60)	−37 ($^{+79}_{-209}$)	0.0 (fixed)	−166 (±58)
K	[m s^{-1}]	0.82 (±0.14)	4.53 (±0.15)	2.92 (±0.16)	4.26 (±0.18)	2.95 (±0.18)	1.55 (±0.22)	3.11 (±0.22)
m sin *i*	[*M*]	1.40 (±0.25)	13.16 (±0.59)	11.91 (±0.75)	25.3 (±1.4)	23.5 (±1.7)	21.3 (±3.2)	65.2 (±4.6)
a	[AU]	0.02226 (±0.00038)	0.0641 (±0.0010)	0.1286 (±0.0021)	0.2695 (±0.0048)	0.4924 (±0.0083)	1.422 (±0.030)	3.40 (±0.12)
N_{meas}					190			
Span	[days]				2428			
rms	[m s^{-1}]				1.27			
$\sqrt{\chi^2}$					1.23			

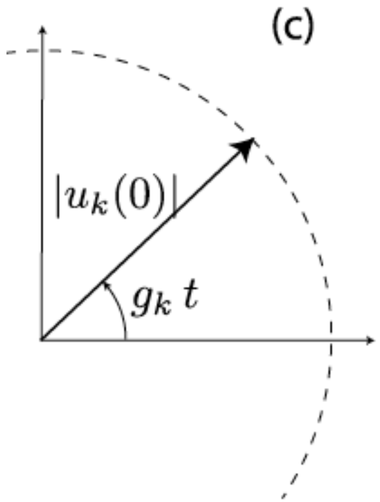
keplerian orbital solution

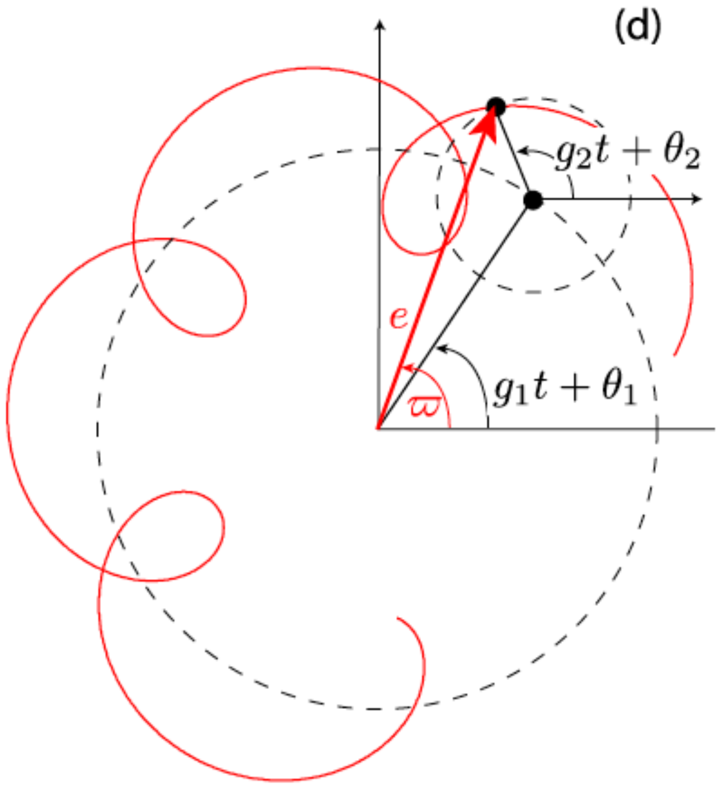
Lovis, Correia, Laskar et al. (A&A 2010)

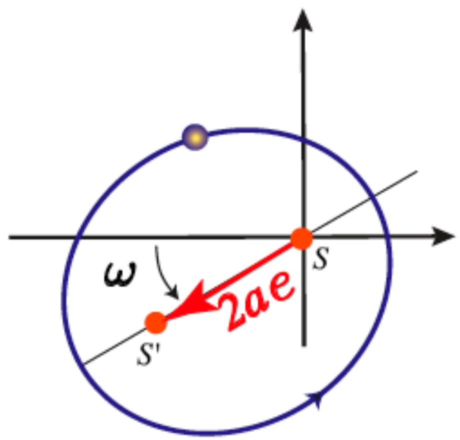
Lagrange-Laplace linear system

$$\begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix} = (S) \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}$$

$$z_k = \sum_j s_{kj} u_j$$

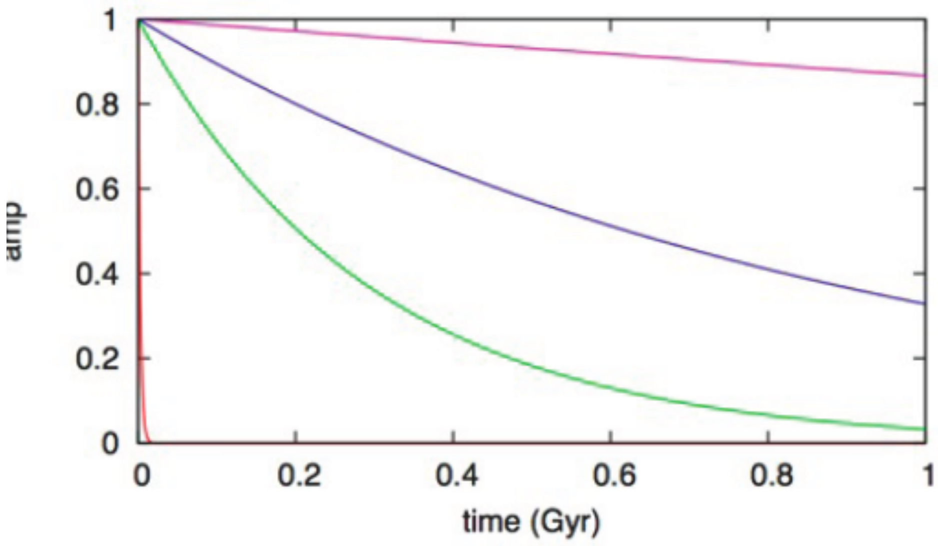






$$z_k = e_k \exp(i\varpi_k)$$

tidal constraint



$$u_k = u_k(0)\mathrm{e}^{-\gamma_k t}\,\mathrm{e}^{\mathrm{i}\,g_k t}$$

1) compute an approximation of (S)

$$\begin{pmatrix} u_1 \\ \vdots \\ u_7 \end{pmatrix} \approx (S)^{-1} \begin{pmatrix} z_b \\ \vdots \\ z_h \end{pmatrix}$$

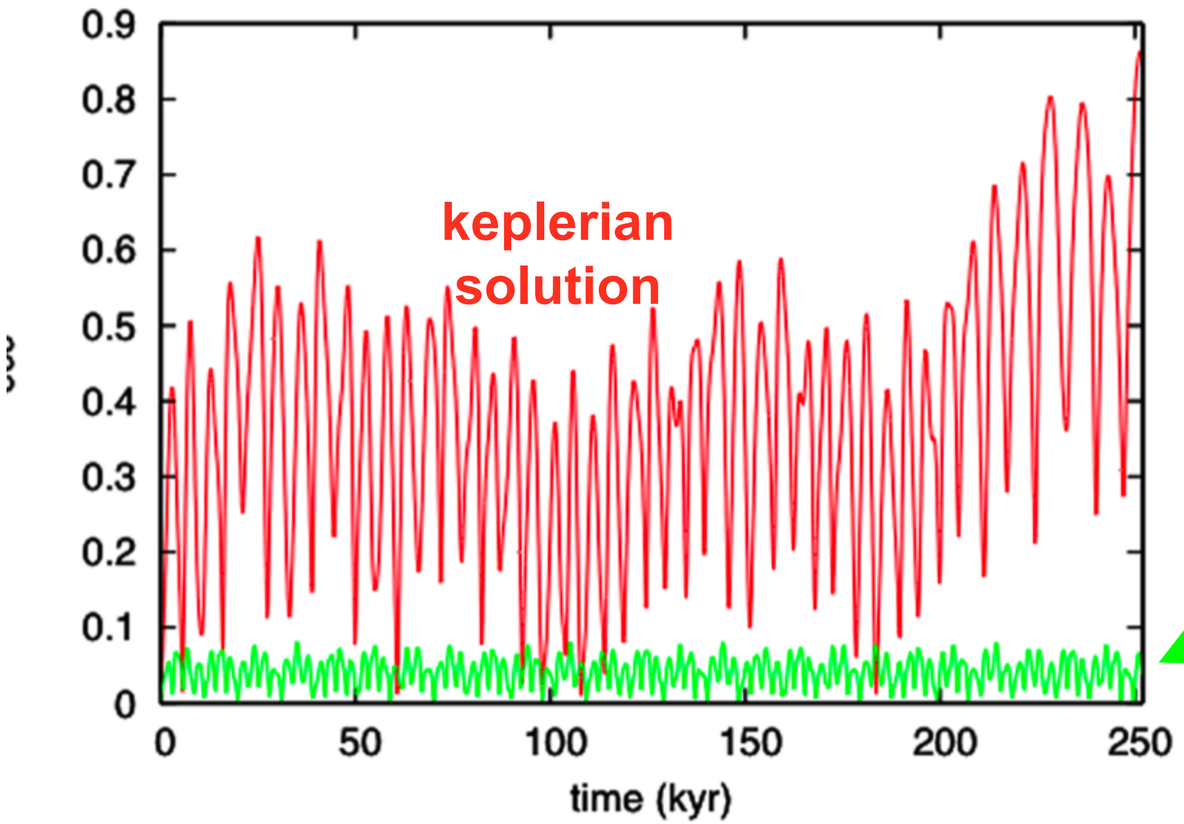
2) compute the u_k

$$u_k = \sum_j S^{-1}_{kj} z_j \approx 0$$

3) add the constraint to χ^2

$$\chi^2_R = R\left(u_1^2 + u_2^2\right)$$

Lovis et al. (A&A 2010) Laskar, Boué, Correia, 2012



non-keplerian orbital solution, with tides

Parameter	[unit]	HD 10180 b	HD 10180 c	HD 10180 d	HD 10180 e	HD 10180 f	HD 10180 g	HD 10180 h
Epoch	[BJD]	2,454,000.0 (fixed)						
	[deg]	90 (fixed)						
V	[km s^{-1}]	35.52981(±0.00012)						
P	[days]	1.17768 (±0.00010)	5.75979 (±0.00062)	16.3579 (±0.0038)	49.745 (±0.022)	122.76 (±0.17)	601.2 (±8.1)	2222 (±91)
	[deg]	188 (±13)	238.5 (±2.3)	196.6 (±3.8)	102.4 (±2.4)	251.2 (±3.6)	321.5 (±9.9)	235.7 (±6.0)
e		0.0000 (±0.0025)	0.045 (±0.026)	0.088 (±0.041)	0.026 (±0.036)	0.135 (±0.046)	0.19 (±0.14)	0.080 (±0.070)
	[deg]	39 (±78)	332 (±43)	315 (±33)	166 (±110)	332 (±20)	347 (±49)	174 (±74)
K	[m s^{-1}]	0.78 (±0.13)	4.50 (±0.12)	2.86 (±0.13)	4.19 (±0.14)	2.98 (±0.15)	1.59 (±0.25)	3.04 (±0.19)
m sin i	[M]	1.35 (±0.23)	13.10 (±0.54)	11.75 (±0.65)	25.1 (±1.2)	23.9 (±1.4)	21.4 (±3.4)	64.4 (±4.6)
a	[AU]	0.02225 (±0.00035)	0.0641 (±0.0010)	0.1286 (±0.0020)	0.2699 (±0.0042)	0.4929 (±0.0078)	1.422 (±0.026)	3.40 (±0.11)
N_{meas}		190						
Span	[days]	2428						
rms	[m s^{-1}]	1.28						

Lovis, Correia, Laskar et al. (A&A 2010)

Conclusions:

- Most of the time, a Keplerian fit is sufficient for the determination of the orbits. In all cases, a Keplerian fit is the first approximation.
- Multi-planet systems are very common, very interesting, but hard to disentangle from observational data.
- Better determinations of the orbital parameters of a system can be achieved when dynamical considerations are taken into account during the fitting procedure.
- For systems that appear to be unstable, specific studies need to be made. Up to now, the solution never simple.
- Radial velocities alone can fully determine the architecture of multi-planet systems without the input from astrometry or transits.
- Dynamical studies of these systems can help the observations when searching for additional planets in the system.